Chapter Seven

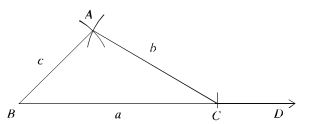
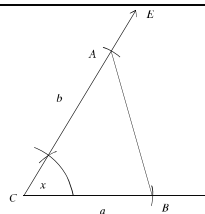
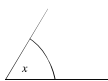
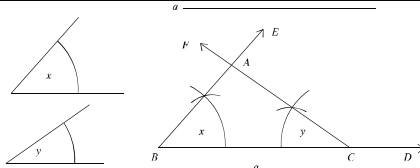
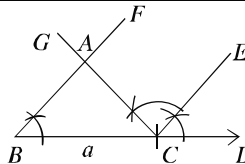
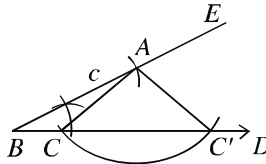
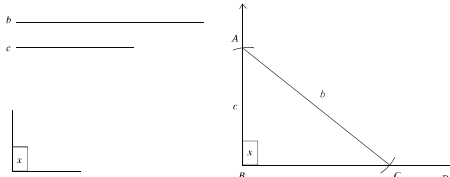
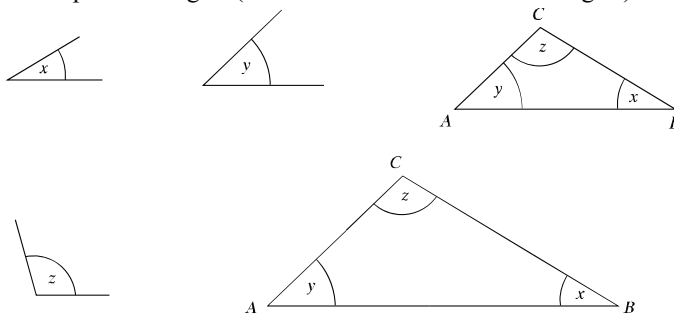
Practical Geometry

In the previous classes geometrical figures were drawn in proving different propositions and in the exercises. There was no need of precision in drawing these figures. But sometimes precision is necessary in geometrical constructions. For example, when an architect makes a design of a house or an engineer draws different parts of a machine, high precision of drawing is required. In such geometrical constructions, one makes use of ruler and compasses only. We have already learnt how to construct triangles and quadrilaterals with the help of ruler and compasses. In this chapter we will discuss the construction of some special triangles and quadrilaterals.

At the end of the chapter, the students will be able to –

- Explain triangles and quadrilaterals with the help of figures
- Construct triangle by using given data
- Construct parallelogram by using given data.

7·1 Construction of Triangles

Every triangle has three sides and three angles. But, to specify the shape and size of a triangle, all sides and angles need not to be specified. For example, as sum of the three angles of a triangle is two right angles, one can easily find the measurement of the third angle when the measurement of the two angles of the triangle given. Again, from the theorems on congruence of triangles it is found that the following combination of three sides and angles are enough to be congruent. That is, a combination of these three parts of a triangle is enough to construct a unique triangle. In class seven we have learnt how to construct triangles from the following data:

Observe that in each of the cases above, three parts of a triangle have been specified. But any three parts do not necessarily specify a unique triangle. As for example, if three angles are specified, infinite numbers of triangles of different sizes can be drawn with the specified angles (which are known as similar triangles).

Sometimes for construction of a triangle three such data are provided by which we can specify the triangle through various drawing. Construction in a few such cases is stated below.

Construction 1

The base of the base adjacent angle and the sum of other two sider of a triangle are given. Construct the triangle.

Let the base a, a base adjacent angle $\angle x$ and the sum s of the other two sides of a triangle ABC be given. It is required to construct it.

Steps of construction :

(1) From any ray BE cut the line segment BC equal to a. At B of the line segment BC, draw an angle $\angle CBF = \angle x$.

(2) Cut a line segment BD equal to s from the ray BF.

(3) Join C,D and at C make an angle $\angle DCG$ equal to $\angle BDC$ on the side of DC in which B lies.

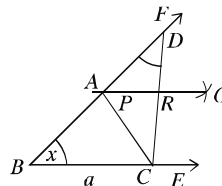
(4) Let the ray CG intersect BD at A.

Then, ABC is the required triangle.

Proof : In ΔACD, $\angle ADC = \angle ACD$ [by construction]

$\therefore \ AC = AD.$

Now, In ΔABC, $\angle ABC = \angle x$, $BC = a$, [by construction]

and $BA + AC = BA + AD = BD = s$. Therefore, ΔABC is the required triangle.

Alternate Method

Let the base a, a base adjacent angle $\angle x$ and the sum s of the other two sides of a triangle ABC be given. It is required to construct the triangle.

Steps of construction:

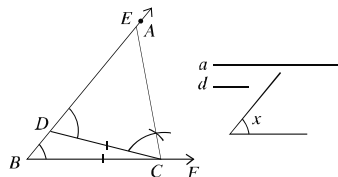
(1) From any ray BE cut the line segment BC equal to a. At B of the line segment BC draw an angle $\angle CBF = \angle x$.

(2) Cut a line segment BD equal to s from the ray BF.

(3) Join C,D and construct the perpendicular bisector PQ of CD.

(4) Let the ray PQ intersect BD at A. Join A, C.

Then, ABC is the required triangle.

Proof: In ΔACR and ΔADR, $CR = DR$ $AR = AR$ and the included angle

$\angle ARC = \angle ARD$ [right angle]

$\Delta ACR \cong \Delta ADR$. $\therefore$ $AC = AD$

Now, In ΔABC, $\angle ABC = \angle x$, $BC = a$, [by construction]

and $BA + AC = BA + AD = BD = s$. Therefore, ΔABC is the required triangle.

Construction 2

The base of a triangle the base adjacent an acute angle and the difference of the other two sides are given. Construct the triangle.

Let the base a, a base adjacent angle $\angle x$ and the difference d of the other two sides of a triangle ABC be given. It is required to construct the triangle.

Steps of Construction :

(1) From any ray BE, cut the line segment BC , equal to a. At B of the line segment BC draw an angle $\angle CBF = \angle x$.

(2) Cut a line segment BD equal to s from the ray BE.

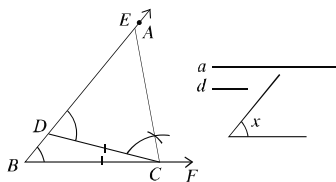
(3) Join C,D and at C, make an angle $\angle DCA$ equal to $\angle EDC$ on the side of DC in which C lies. Let the ray CA intersect BE at A.

Then ABC is the required triangle.

Proof :, In ΔACD, $\angle ADC = \angle ACD$ [by construction]

$\therefore$ $AC = AD$.

So, the difference of two sides $AB - AC = AB - AD = BD = d$.

Now, In ΔABC, $BC = a, AB - AC = d$ and $\angle ABC = \angle x$. Therefore, ΔABC is the required triangle.

Activity :

1. If the given angle is not acute, the above construction is not possible. Why ? Explore any way for the construction of the triangle under such circumstances.
2. The base, the base adjacent angle and the difference of the other two sides of a triangle are given. Construct the triangle in an alternate method.

Construction 3

The base adjacent two angles and the perimeter of a triangle are given. Construct the triangle.

Let the base adjacent angles $\angle x$ and $\angle y$ and the perimeter p be geven. It is required to construct the triangle.

Steps of Construction :

(1) From any ray DF, cut the part DE equal to the perimeter p. Make angles $\angle EDL$ equal to $\angle x$ and $\angle DEM$ equal to $\angle y$ on the same side of the line segment DE at D and E.

(2) Draw the bisectors BG and EH of the two angles.

(3) Let these bisectors DG and EH intersect at a point A. At the point A, draw $\angle DAB$ equal to $\angle ADE$ and $\angle EAC$ equal to $\angle AED$.

(4) Let AB intersect DE at B and AC intersect DE at C.

Then, ΔABC is the required triangle.

Proof : In ΔADB, $\angle ADB = \angle DAB$ [by construction] $\therefore AB = DB$.

Again, in ΔACE, $\angle AEC = \angle EAC$; $\therefore CA = CE$.

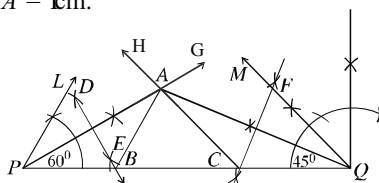
Therefore, in ΔABC, $AB + BC + CA = DB + BC + CE = DE = p$.

$$\angle ABC = \angle ADB + \angle DAB = \frac{1}{2}\angle x + \frac{1}{2}\angle x = \angle x$$

and $\angle ACB = \angle AEC + \angle EAC = \frac{1}{2}\angle y + \frac{1}{2}\angle y = \angle y$. Therefore, ΔABC is the required triangle.

Activity:

1. Two acute base adjacent angles and the perimeter of a triangle are given. Construct the triangle in an alternative way.

Example 1. Construct a triangle ABC, in which $\angle B = 60^\circ$, $\angle C = 45^\circ$ and the perimeter $AB + BC + CA = 1$cm.

Steps of Construction: Follow the steps below :

(1) Draw a line segment $PQ = 1$cm.

(2) At P, construct an angle of $\angle QPL = 60^\circ$ and at Q an angle of $\angle PQM = 45^\circ$ on the same side of PQ.

(3) Draw the bisectors PG and QH of the two angles. Let the bisectors PG and QH of these angles intersect at A.

(4) Draw perpendicular bisector of the segments PA of QA to intersect PQ at B and C.

(5) Join *A, B* and *A, C*.
Then, *ABC* is the required triangle.

Activity : An adjacent side with the right angle and the difference of hypotenuse and the other side of a right-angled triangle are given. Construct the triangle.

Exercise 7.1

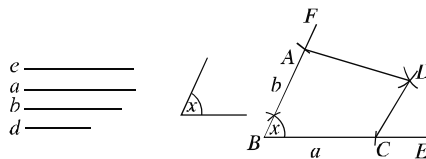
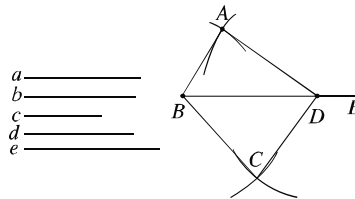
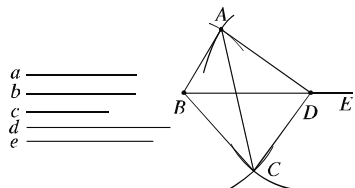
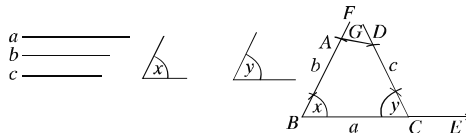
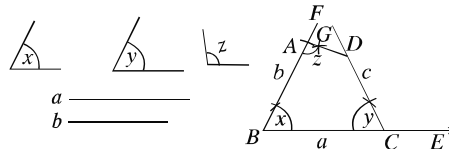
1. **Construct a triangle with the following data:**
   (a) The lengths of three sides are 3 cm, 3·5 cm, 2·8 cm.
   (b) The lengths of two sides are 4 cm , 3 cm and the included angle is 60°.
   (c) Two angles are 60° and 45° and their included side is 5 cm.
   (d) Two angles are 60° and 45° and the side opposite the angle 45° is 5 cm.
   (e) The lengths of two sides are 4.5 cm and 3.5 respectively and the angle opposite to the second side is 4 cm.
   (f) The lengths of the hypotenuse and a side are 6 cm and 4 cm respectively.
2. **Construct a triangle *ABC* with the following data:**
   (a) Base 3.5 cm, base adjacent angle 60° and the sum of the two other sides 8 cm.
   (b) Base 4 cm, base adjacent angle 50° and the sum of the two other sides 7.5 cm.
   (c) Base 4 cm, base adjacent angle 50° and the difference of the two other sides 1.5 cm.
   (d) Base 5 cm, base adjacent angle 45° and the difference of the two other sides 1 cm.
   (e) Base adjacent angles 60° and 45° and the perimeter 12 cm.
   (f) Base adjacent angles 30° and 45° and the perimeter 10 cm.
3. Construct a triangle when the two base adjacent angles and the length of the perpendicular from the vertex to the base are given.
4. Construct a right-angled triangle when the hypotenuse and the sum of the other two sides are given.
5. Construct a triangle when a base adjacent angle, the altitude and the sum of the other two sides are given.
6. Construct an equilateral triangle whose perimeter is given.
7. The base, an obtuse base adjacent angle and the difference of the other two sides of a trangle are given. Construcet the triangle.

7·2 Construction of Quadrilaterals

We have seen if three independent data are given, in many cases it is possible to construct a definite triangle. But with four given sides the construction of a definite quadrilateral is not possible. Five independent data are required for construction of a definite quadrilateral. A definite quadrilateral can be constructed if any one of the following combinations of data is known :

(a) Four sides and an angle
(b) Four sides and a diagonal
(c) Three sides and two diagonals
(d) Three sides and two included angles
(e) Two sides and three angles.

In class VII, the construction of quadrilaterals with the above specified data has been discussed. If we closely look at the steps of construction, we see that in some cases it is possible to construct the quadrilaterals directly. In some cases, the construction is done by constructions of triangles. Since a diagonal divides the quadrilateral into two triangles, when one or two diagonals are included in data, construction of quadrilaterals is possible through construction of triangle.

(1) Four sides and an angle

(2) Four sides and a diagonal

(3) Three sides and two diagonals

(4) Three sides and two included angles

(5) Two sides and three angles

Sometimes special quadrilaterals can be constructed with fewer data. In such a cases, from the properties of quadrilaterals, we can retrieve five necessary data. For example, a parallelogram can be constructed if only the two adjacent sides and the included angle are given. In this case, only three data are given. Again, a square can be constructed when only one side of the square is given. The four sides of a square are equal and an angle is a right angle; so five data are easily specified.

Construction 4

Two diagonals and an included angle between them of a parallelogram are given. Construct the parallelogram.

Let a and b be the diagonals of a parallelogram and $\angle x$ be an angle included between them. The parallelogram is to be constructed.

Steps of construction:

From any ray AE, cut the line segment $AC = a$. Bisect the line segment AC to find the midpoint O, At O construct the angle $\angle AOP = \angle x$ and extend the ray PO to the opposite ray OQ. From the rays OP and OQ cut two line segments OB and OD equal to $\frac{1}{2}$ b. Join $A,B;$ $A,D;$ C,B and C,D. Then $ABCD$ is the required parallelogram.

Proof: In triangles ΔAOB and ΔCOD,

$OA = OC = \frac{1}{2}a,\ OB = OD = \frac{1}{2}b$ [by construction]

and included $\angle AOB$ = included $\angle COD$ [opposite angle]

Therefore, $\Delta AOB \cong \Delta COD$.

So, $AB = CD$

and $\angle ABO = \angle CDO$;but the two angles are alternate angles.

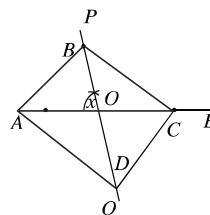
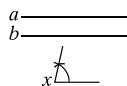
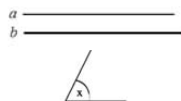
$\therefore$ AB and CD are parallel and equal.

Similarly, AD and BC are parallel and equal.

Therefore, $ABCD$ is a parallelogram with diagonals

$AC = AO + OC = \frac{1}{2}a + \frac{1}{2}a = a$

and $BD = BO + OD = \frac{1}{2}b + \frac{1}{2}b = b$ and the angle included between the diagonals $\angle AOB = \angle x$.

Therefore, $ABCD$ is the required parallelogram.

Construction 5

Two diagonals and a side of a parallelogram are given. Construct the parallelogram.

Let a and b be the diagonals and c be a side of the parallelogram. The parallelogram is to be constructed.

Steps of construction:

Bisect the diagonals a and b to locate their midpoints. From any ray AX, cut the line segment $AB = a$. With centre at A and B draw two arcs with radii $\frac{2}{a}$ and $\frac{b}{2}$ respectively on the same side of AB. Let the arcs intersect at O. Join A, O and O, B. Extend AO and BO to AE and BF respectively. Now cut $OC = \frac{2}{a}$ and $OD = \frac{b}{2}$ from OE and OF respectively. Join A,D; D,C; C,B. Then $ABCD$ is the required parallelogram.

Proof: In ΔAOB and ΔCOD,

$OA = OC = \frac{a}{2}; OB = OD = \frac{b}{2}$, [by construction]

and included $\angle AOB$ = included $\angle COD$ [opposite angle]

$\therefore \quad \Delta AOB \cong \Delta COD$.

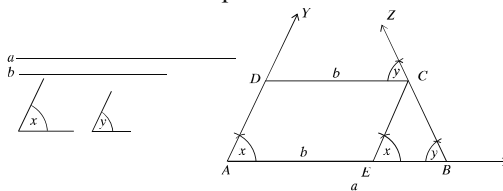
$\therefore \quad AB = CD$ and $\angle ABO = \angle ODC$; but the angles are alternate angles.

$\therefore \quad AB$ and CD are parallel and equal.

Similarly, AD and BC are parallel and equal.

Therefore, $ABCD$ is the required parallelogram.

Example 1. The parallel sides and two angles included with the larger side of a trapezium are given. Construct the trapezium.

Let a and b be the parallel sides of a trapezium where $a > b$ and $\angle x$ *and* $\angle y$ be two angles included with the side a. The trapezium is to be constructed.

Steps of construction:

From any ray *AX,* cut the line segment $AB = a$. At A of the line segment AB, construct the angle $\angle BAY = \angle x$ and at B, construct the angle $\angle ABZ = \angle y$. From the line segment AB, cut a line segment $AE=b$. Now at E, construct $BC \parallel AY$ which cuts BZ at C. Now construct $CD \parallel BA$. The line segment CD intersects the ray AY at D. Then $ABCD$ is the required trapezium.

Proof : By construction, $AB \parallel CD$ and $AD \parallel EC$. Therefore, $AECD$ is a parallelogram and $CD = AE = b$. Now in the quadrilateral $ABCD$, $AB=a$, $CD=b$, $AB \parallel CD$ and $\angle BAD = \angle x$, $\angle ABC = \angle y$ (by construction). Therefore, $ABCD$ is the required trapezium.

Activity : The perimeter and an angle of a rhombus are given. Construct the rhombus.

Exercise 7·2

1. The two angles of a right angled triangle are given. Which one of the following combination allows constructing the triangle?

   a. 63° and 36° b. 30° and 70° c. 40° and 50° d. 60° and 20°

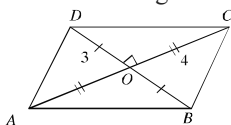
2. i. A rectangle is a parallelogram

   ii. A square is a rectangle

   iii. A rhombus is a square

   On the basis of the above information, which one of the following is true?

   a. i and ii b. i and iii c. ii and iii d. i, ii and iii

   In view of the given figure, answer the questions 3 and 4.

3. What is the area of ΔAOB?

   a. 6 sq. units b. 7 sq. units c. 12 sq. units d. 14 sq. units

4. The perimeter of the quadrilateral is

   a. 12 units b. 14 units c. 20 units d. 28 units

5. **Construct a quadrilateral with the following data :**

   (a) The lengths of four sides are 3 cm, 3·5 cm, 2·5 cm, 3cm and an angle is 45°.

   (b) The lengths of four sides are 3.5 cm, 4 cm, 2·5 cm 3.5 cm and a diagonal is 5 cm.

   (c) The lengths of three sides are 3·2 cm, 3 cm, 3·5 cm and two diagonals are 2·8 cm, and 4.5 cm.

6. Construct a parallelogram with the following data:
   (a) The lengths of two diagonals are 4 cm, 6·5 cm and the included angle is $45°$.
   (b) The lengths of two diagonals are 5 cm, 6·5 cm and the included angle is $30°$.
   (c) The length of a side is 4 cm and the lengths of two diagonals are 5 cm and 6·5 cm.
   (d) The length of a side is 5 cm and the lengths of two diagonals are 4.5 cm and 6 cm.
7. The sides AB and BC and the angles $\angle B$, $\angle C$, $\angle D$ of the quadrilateral $ABCD$ are given. Construct the quadrilateral.
8. The four segments made by the intersecting points of the diagonals of a parallelogram and an included angle between them are $OA = 4$ cm. $OB = 5$ cm. $OC = 3\cdot5$ cm $OD = 4\cdot5$ cm and $\angle AOB = 80°$ respectively. Construct the quadrilateral.
9. The length of a side of a rhombus and an angle are 3.5 cm. and $45°$ respectively; construct the rhombus.
10. The length of a side and a diagonal of a rhombus are given; construct the rhombus.
11. The length of two diagonals of a rhombus are given. Construct the rhombus.
12. The perimeter of a square is given. Construct the square.
13. The houses of Mr. Zoki and Mr. Zafrul are in the same boundary and the area of their house is equal. But the house of Zoki is rectangular and the house of Mr. Zafrul is in shape of parallelogram.
   (a) Construct the boundary of each of their houses taking the length of base 10 units and height 8 units.
   (b) Show that the area of the house of Mr. Zoki is less than the area of the house of Mr. Zafrul.
   (c) If the ratio of the length and the breadth of the house of Mr. Zoki is 4:3 and its area is 300 sq. units, find the ratio of the area of their houses.
14. The lengths of the hypotenuse and a side of right angled triangle are 7 cm and 4 cm. Use the information to answer the following questions :
   a. Find the length of the other side of the triangle.
   b. Construct the triangle.
   c. Construct a square whose perimeter is equal to the perimeter of the triangle.
15. $AB = 4$ cm, $BC = 5$ cm $\angle A = 85^{o}$, $\angle B = 80^{o}$ and $\angle C = 95^{o}$. of the quadrilateral $ABCD$. Use the information to answer the following questions:
   a. Construct a rhombus and give the name.
   b. Use the above information to construct the quadrilateral $ABCD$.
   c. Construct an equilateral triangle whose perimeter is equal to the perimeter of the quadrilateral $ABCD$.